Do not forget the cancellation

Marking-to-market and hedging LCDX tranches

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Although market is busy today working on the bullet LCDS contract to remove the cancellation feature from syndicated secured loan derivatives, in their current form LCDSs and LCDX tranches are still exposed to the cancellation risk. Until recently, in lack of proper modelling framework, market practitioners neglected the cancellation risk and they priced and hedged these products as simple CDSs and CDO tranches. However, cancellation risk does matter! Especially in the current market situation. As we show here, it is more than important to take into account the cancellation risk while marking-to-market and hedging syndicated secured loan derivatives. For this purpose, we present here an easy and robust way to model the cancellation.

1 Introduction

After months of discussions, by September 2008 the dealer group has drafted a new bullet form of LCDS contracts and submitted it to ISDA for approval. This new form removes the cancellation language, so the contract stays in place even if all of the bank debt goes away. From this new contract they expect the increase of liquidity on the market, since they focused on removing the cancellation feature that makes pricing difficult and idiosyncratic and has contributed to illiquidity in the market for the derivative. Removing the cancellation option should have the effect of standardizing unwinds and, as a result, valuation.

However, not everybody has welcomed the new contract. Portfolio managers that manage loan risk prefer the old contract, because in case the underlying loan went away, the LCDS on the loan would also go away. Furthermore, some traders worry about, that even if the LCDS is not cancelled, the cancellation of the underlying loan may have a gamma effect and the volatility of basket expected loss may change too. Therefore, some risks related to cancellation will still remain. An other aspect is what will happen with the existing old contracts. Even introducing the new bullet LCDS contract, the rolling of liquid names from old to new contracts will depend on the value of cancellation option and it is even not clear how long the rolling may last.

In September 2008, market players hoped to have the bullet contract put in place before the New Year, but still there is no time for the launch, which is definitively not in sight at the moment. However, institutional investors and portfolio managers do have LCDSs and LCDX tranches in their books that should be revaluated and hedged on a day-to-day basis. In this paper, focusing on the old contract we give simple and robust techniques to take cancellation risk into account.

2 Modelling the LCDS spreads

At first, we need to identify trigger events (like default, cancellation or restructuring) that can change the corporate status (like defaulted, cancelled or survived). The corporate status is modelled by a continuous time Markov chain, where the change of the status is defined by the default and prepayment intensities (correspondingly $\lambda_d(t)$ and $\lambda_p(t)$). This specifies a common reduced-form model for credits. It follows that

$$
\begin{aligned}
P_{survived}(\tau) &= E\left[\exp\left(-\int_0^\tau \left(a\lambda_p(u)+\lambda_d(u)\right)du\right)\right]\\
P_{cancelled}(\tau) &= E\left[\int_0^\tau a\lambda_p(s)\exp\left(-\int_0^s \left(a\lambda_p(u)+\lambda_d(u)\right)du\right)ds\right]\\
P_{defaulted}(\tau) &= E\left[\int_0^\tau \lambda_d(s)\exp\left(-\int_0^s \left(a\lambda_p(u)+\lambda_d(u)\right)du\right)ds\right]
\end{aligned}
$$

By construction the sum of these three probabilities is equal to one for each maturity. The parameter a is used to model the efficiency of substitution. Let us say, for a current non-cancellable contract in case of prepayment a substituter loan is found with 90% probability, thus $a = 0.1$ showing that the contract is less exposed to cancellation risk than a cancellable counterpart. This implies, that for the same issuer, same seniority and restructuring clause the above conditional default probabilities are implicitly higher for non-cancellable LCDS contracts than for cancellable LCDS contracts. This happens, because the cancellation risk amortizes both the default and survival probabilities.

3 Moments and the role of cancellation

Until now we made no restrictions on the introduced reduced-form model. The underlying intensity processes can be deterministic or stochastic, there can be jumps or no jumps, the intensities can be correlated or not. However, all these factors have impact more on the higher order moments of the cumulated intensities. Therefore, the proper choice of the intensity processes may be important to price LCDS options and other sophisticated products, but calculating only the above probabilities and fair spreads of LCDSs, it is the first moment that matters. As investigated in [6], this is also the reason why neither the correlated default and prepayment intensities nor the correlated stochastic recoveries can explain the joint puzzle of CDS and LCDS spreads.

Considering the pricing of LCDSs, the cancellation risk has double effect. The cancellation probability is amortizing both the default leg and the fee leg of an LCDS. Therefore, as detailed in [6], the impact of the cancellation probability on the fair spread is not significant. This explains why – in spite of some market beliefs – implied prepayment probabilities can hardly be derived solely from CDS and LCDS spreads. However, the cancellation probability indeed plays an important role in marking-to-market and managing the risks of an outstanding LCDS position. Due to possible cancellation the risky annuity of the LCDS as a measure for the expected maturity is definitively lower than it would be when disallowing cancellation.

The price of LCDX tranches just like the price of classical CDO tranches are more first moment derivatives and they are less exposed to single name risks like stochastic volatility of fair spreads or jumps in the intensity processes. Therefore, seeking for efficient hedging and marking-to-market of LCDS tranches implies less focus on the processes of the underlying intensities and implies more need to focus on the robust estimation of prepayment probabilities. To price LCDX tranches we may consider to use even a simplistic LCDS model with deterministic default and prepayment intensities. We can do so, because, as we show later, the only information we need from the single name LCDS model are the conditional probabilities defined above and some further recovery assumptions.

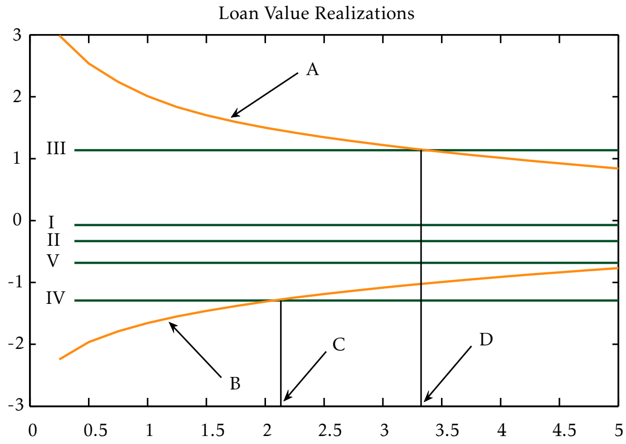
Concerning the calibration to single names, once we have assumptions about the prepayment intensities, we bootstrap the default intensities from LCDS spreads as it is usually done when dealing with simple CDSs. Since we are not able to get implied cancellation rates from LCDS quotes, working with large homogeneous portfolio models we will assume a constant cancellation rate being homogeneous for each single name contract in the portfolio.

4 Extended base correlation models

Although base correlation models for CDOs as introduced in [9] are often criticized, with their extensions like Lévy copulas ([1], [4], [8]) and stochastic recovery ([2]) they are still in use on the market because of their simplicity and practicality. These models are definitively not reliable to price forward CDOs or index options, but they are robust enough for marking-to-market and hedging CDO tranches. Until recently, the base correlation models exhibited the default and stochastic recovery features, but not the cancellation one. However, as we show here, these one-factor models can be easily extended to incorporate cancellation.

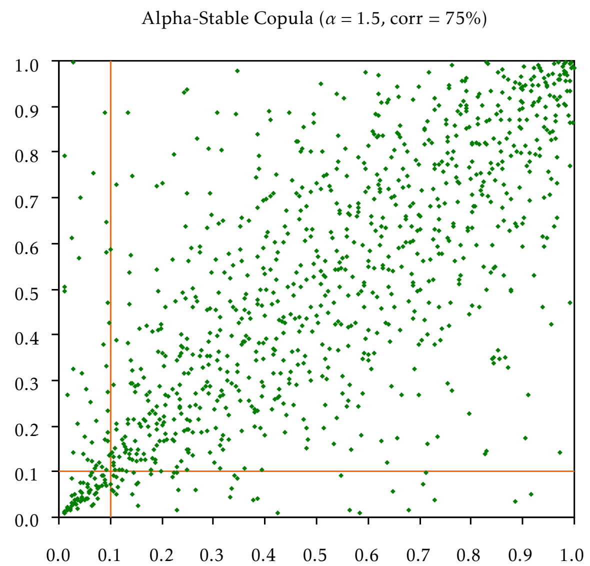
In the one-factor models the common factor y plays the role of the market. In our extended model we assume that when the factor is high, the conditional cancellation probabilities $q(y;t)$ that are high, when it is low, the conditional default probabilities $p(y;t)$ that are high. This seems to be a realistic assumption. In downturn the companies are more likely to default, while in good health the companies are more likely to repay their loans and apply for new ones but with lower rates.

On the following figure we present how the default and cancellation time distributions are correlated. Based on the Gaussian copula with correlation 30% and assuming that the issuers have the same default and cancellation probabilities – thus they have the same default and cancellation barriers, – we show five loan value realizations. In the simulated scenario loan III prepays before $T = 5$, loan IV defaults before $T = 5$ and all the others run until maturity $T = 5$.



Horizontal lines are different loans, A (cancellation barrier), B (default barrier), C (default time of loan IV), D (cancellation time of loan III)

Unfortunately, in this one-factor model the Gaussian copula correlates the default time distributions and the cancellation time distributions equally. However, one may argue that in downturn the correlation between defaults are usually higher than the correlation between cancellations in upturn. Therefore, it may be reasonable to use a skewed Lévy copula like the α-stable copula, which allows different correlations on the tails of the common factor's distribution. Using such a copula, we may approach the Armageddon scenario on the default tail, while not allowing cancellations in large number on the other tail. Thanks to this trick, while being intuitive the model remains tractable with only a few parameters (the α-stable copula has only one parameter more than the Gaussian copula).



Concerning the correlation between defaults and prepayments, in line with our intuition, it is negative. However, – though one may think – the one-factor model does not mean perfect negative correlation between defaults and prepayments. Let us consider a scenario, where the simulated market factor y is very low. In this case the default probabilities conditional on the market state $p_i(y;t)$ are very high. But, the default of the best issuer in a scenario does not mean that all the others will default too. Although the likelihood is very low, mathematically the probability is still there that the worst issuer will repay its loan (or merge with an other company that is triggered as cancellation too).

Now, considering the mathematical formulation of the proposed model, the major difference compared to well-known one-factor models is that we allow the possibility of cancellation too. In order to obtain the conditional loss distribution function $\Pi^y_{k,l}(t)$, we apply a recursive formula as in [3], but we extend it with the conditional cancellation probabilities $q_i(y;t)$. We mark the extension with colours.

$$\begin{aligned}\Pi^{y,n+1}_{k,l}(t) &= \left(1-p_{n+1}(y;t)-q_{n+1}(y;t)\right)\Pi^{y,n}_{k,l}(t)\\ &+ p_{n+1}(y;t)\,\Pi^{y,n}_{k-1,l}(t)+q_{n+1}(y;t)\,\Pi^{y,n}_{k,l-1}(t)\end{aligned}$$

Due to the extension the recursive formula will not produce here a vector, but a triangular matrix. The elements of the matrix will tell us what the probability of scenarios are where k companies go to default and l companies repay its loan triggering a cancellation of the LCDS. The matrix is triangular, because a scenario where the number of defaults added to the number of cancellations is higher than the number of issuers is impossible.

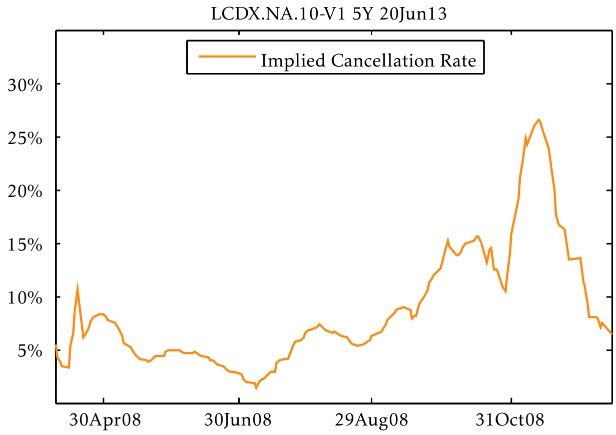
The joint conditional density and marginal distributions are calculated as usual. In order to price the credit leg of a tranche, we need to calculate the expected loss on the tranche, while to revalue the fee leg of the tranche, we have to calculate the expected notional of the tranche. The difference compared to common one-factor models as those in [1] is that the notional may be decreased not only by defaults, but also by cancellations that from pricing point of view are considered as defaults with recovery of 100%. For the whole portfolio the expected loss $E[L(t)]$ and the expected notional decrease $E[\Delta N(t)]$ are calculated as follows.

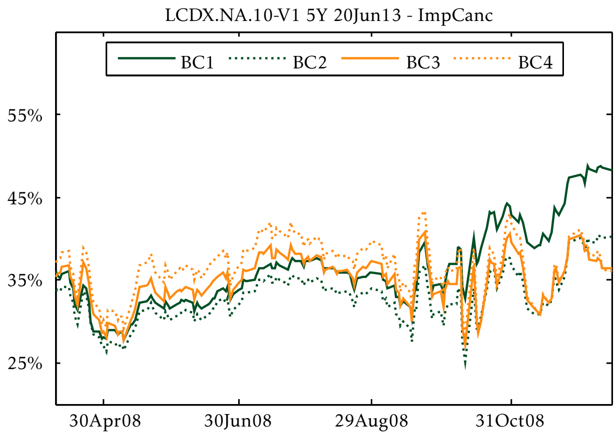
$$\begin{aligned}E[L(t)] &= \int_y \left(\left(1-\bar{R}(y)\right)\cdot\sum_{k=1}^{m}\frac{k}{m}\sum_{l=0}^{m-k}\Pi^y_{k,l}(t)\right)\mathrm{d}\mathcal{H}_\rho(y)\\ E[\Delta N(t)] &= \int_y \left(\sum_{l=1}^{m}\frac{l}{m}\sum_{k=0}^{m-l}\Pi^y_{k,l}(t)+\sum_{k=1}^{m}\frac{k}{m}\sum_{l=0}^{m-k}\Pi^y_{k,l}(t)\right)\mathrm{d}\mathcal{H}_\rho(y)\end{aligned}$$

where $\mathcal{H}_\rho(y)$ is the distribution function of the common factor y. A more detailed description of the mathematical formulation can be found in [7]. Concerning the recovery rates $R(y)$ in the expected loss formula, as introduced in [2] we assume that they are deterministic functions of the copula factor y. In this manner, the recovery rates are stochastic and correlated with the market factor. Concerning loan-only credit indices, the extension of the one-factor model with stochastic recovery is not only intuitive, but it is required in all cases to be able to match the tranche quotes. Otherwise, the base correlations would hit the barrier of 100%.

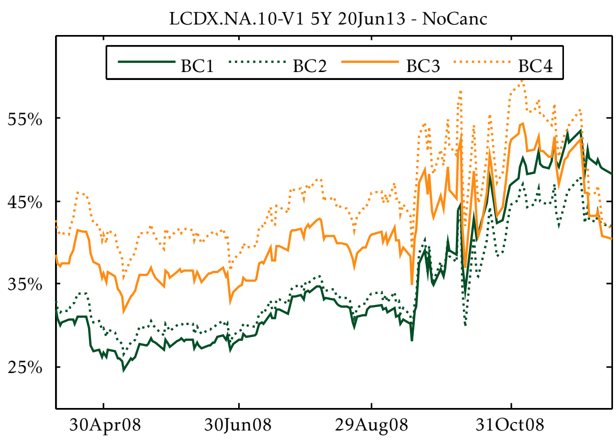
5 How do we calibrate the cancellation rate?

The remaining question before we can use our extended base correlation model is how we can calibrate the cancellation rate. The spread of the super senior tranche is a redundant parameter in the base correlation model. Normally, the CDO like models fail to match the market quotes for the LCDX super senior tranche. This already suggests that some important factors like the cancellation are definitively missing from the pure CDO models. Using the LCDX super senior quotes to find implied cancellation rates, the analysis for the last roll of LCDX follows. The configuration is: α-stable Lévy copula with characteristic 1.9 and stochastic recovery with market correlation of 90%.





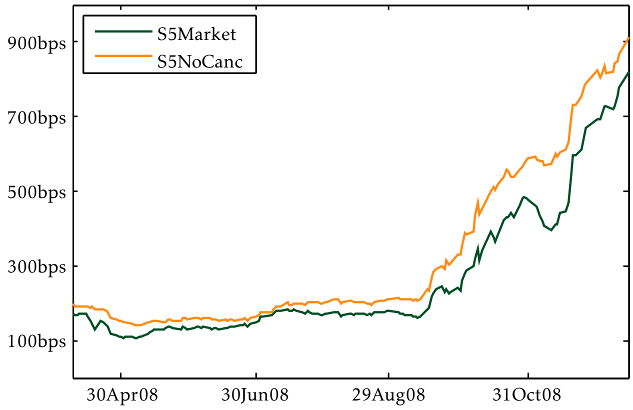
BC1 to BC4 note the base correlations related to the first four tranches of LCDX. For comparison, we present also the calibration of a pure CDO model. The configuration for the comparison is: Gaussian copula and stochastic recovery with market correlation of 90%, but without the cancellation feature.



Comparing the two figures we may conclude that including cancellation into the one-factor model, the base correlation curve is flatter and more stable in time. In both cases for some days we may recognise base correlation smiles instead of base correlation skew. Although it may be unusual – also for traditional CDOs the smile has appeared only recently, – the smile causes no pricing problem as long as the base correlation curve is close to be flat like in this case.

Furthermore, one may argue that these implied cancellation rates do not meet some expectations. Obviously, an expert may have his own cancellation rate that may periodically deviate from the cancellation rates implied by this model. The implied cancellation rate is an indication that assures a fit to the super senior tranche spread too, but beside that one may calculate risky annuities and deltas with other rates too. It does not limit the application of the model. For comparison, we show below how the one-factor model without cancellation can fit the super senior tranche quotes. The configuration is as earlier: Gaussian copula and stochastic recovery with market correlation of 90%.

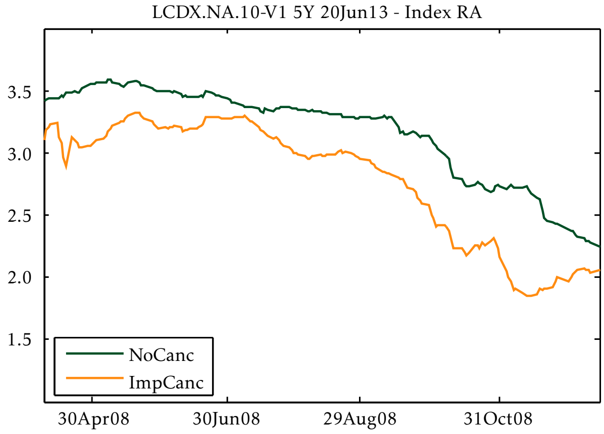
LCDX.NA.10-V1 5Y 20Jun13 - Super Senior Spreads



6 Marking-to-market

The daily revaluation and unwind of single name, index and tranche positions require the estimation of the so-called risky annuity. Opposite to the delta, the risky annuity is not directly observable on the market. Therefore, the daily revaluation and possible unwind may be cumbersome. After we defined the mathematical formulation for the expect loss and expected notional decrease, even with the extension to cancellation the calculation of risky annuities for tranches remains the same job as for the common one-factor models.

LCDX.NA.10-V1 5Y 20Jun13 - Index RA



As explained earlier, the cancellation decreases the risky annuities. In this manner, the cancellation assumption has a significant effect on the MtM of outstanding positions. Looking at the risky annuities calculated for the LCDX, which are also the weighted sum of the tranches' risky annuities, we may conclude that in some cases disregarding from cancellation may cause a difference in MtM of 47%!

7 Hedging

In case of credit derivatives it is quite difficult to prove that a model implies proper hedging ratios. Therefore, this kind of hedge tests are usually missing from articles in the literature. Nevertheless, in this section we compare the hedging performance of the one-factor model with and without cancellation. Obviously, if we would test the two versions separately, we would not see much differences in hedging performance, because the deltas and risky annuities of the same model are always consistent with each other. We see difference only if we evaluate the delta of one model with the risky annuity as measure of the other model.

Since our main hypothesis is that cancellation should be taken into account, we use the risky annuities from the model with cancellation to measure the daily PnLs and we test how the deltas produced by the two different models (with and without cancellations) perform comparing to each other. For this purpose, we apply the following strategy every day in the service of the hedge test.

Day 0: Sell protection via LCDX tranche, notional of 1M USD (receive upfront and spread)
Buy protection via the index, notional given by the hedge ratio, delta (pay spread)

Day 1: Close the positions

As a measure for comparison, we computed the standard deviation of the daily PnLs. Although a pure index hedge against tranches is never satisfactory – hedge by constituents are definitively more efficient, – in line with our expectations we concluded that the model with implied cancellation rate performs better than the model without cancellation. The difference between the two ways of hedging is not enormous, but consistent and shows that cancellation risk does matter! This analysis considered only the simple index hedge, but carrying out the hedge by single names, the impact would be more pronounced.

Standard deviations of the daily PnL using index hedge

	0%-5%	5%-8%	8%-12%	12%-15%	15%-
NoCanc	6,179$	12,229$	21,405$	17,269$	7,910$
ImpCanc	6,181$	12,103$	21,090$	16,914$	6,687$

The difference is most obvious when we look at the super senior tranche, which is the tranche most exposed to the cancellation risk. Analysing this tranche, we found that applying the model with implied cancellation the distribution of the daily PnLs has smaller standard deviation and the distribution is less skewed than it would be disallowing cancellation.

An important aspect is how the cancellation risk can be hedged. Based on [6], the cancellation rate has no significant impact on LCDS spreads. Therefore, it has no impact on the LCDX index spread neither. In this manner, the cancellation risk of LCDX tranches cannot be hedged by the index or by constituents. Only the default risk can be hedged by them.

8 Conclusion

In this paper we introduced a simple and robust way to handle the cancellation feature, which is present in the LCDS and LCDX tranche contracts. Analysing pricing and risk managing perspectives we showed that taking into account the cancellation risk while marking-to-market and hedging syndicated secured loan derivatives is more than important. Therefore, institutional investors and portfolio managers keeping LCDSs and LCDX tranches in their books should consider the revaluation and hedge of those products by incorporating also the cancellation into their modelling framework.

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